REMARKS

Reconsideration and allowance are respectfully requested.

The Applicants thank the Examiner for her efforts in examining this application to date.

Claim 1 is the only independent claim. Claims 1-4, 6, 8-13, 17-19, 22-26, and 30-33 are pending. Claims 1 and 23-24 are currently amended. Exemplary support for these amendments is expressly provided in the original specification at, for examples, ¶¶ 12, 14, 25, 32, 47, 50, and Fig. 9. In addition, inherent support may also be present. No new matter is believed to be added.

INDEPENDENT CLAIM 1 IS NOVEL: REJECTION UNDER 35 USC § 102

The Examiner rejects claims 1-4, 6, 8-13, 17-19, 22-23, 25-26, and 30-33 under 35 USC § 102(e) as being anticipated by Lazar et al. (US Patent Publ. No. 2004/0208751). The Applicants respectfully traverse, particularly in view of the present Amendment.

Independent claim 1, as amended, includes a structural limitation of a fluid inside the chamber, wherein the fluid is in contact with the electrodes, and wherein the fluid comprises a gas phase and a liquid phase. Because Lazar does not teach nor enable such a two-phase fluid, it cannot anticipate claim 1.

Lazar's disclosure is limited to single-phase fluids. Lazar teaches a fluid, being either a liquid or a gas. Lazar at ¶ 6. The motive force for Lazar's pump is electroosmotic pressure generation, rather than generation of gas by electrochemical reaction. Id. at ¶ 28. But, as is discussed below, Lazar's detailed description of electroosmotic pressure generation relies on single-phase fluid mechanics. Id. at ¶¶ 49-61. Moreover, none of Lazar's figures nor remaining text depict or discuss multi-phase fluids.

Lazar's working examples enable only single-phase fluids. The examples rely on Fig 4B, which was constructed using Eqns. (1)-(4). Lazar at ¶¶ 52, 54, 66-67 (accounting for the obvious typographical errors in ¶¶ 66-67, where Fig. 4B is referred to as Fig. 4A). But Eqns. (1)-(2) are based on Poisseuille's law. *Id.* at ¶¶ 49-50. Poisseuille's law assumes that flow is laminar and incompressible, that the fluid is Newtonian, and that fluid density and viscosity are uniform everywhere in a completely-filled circular duct. Poisseuille's law does not describe multi-phase

gas-liquid flow. (Compare treatment of liquid laminar flow in filled circular ducts to that of gas-liquid flow in Chemical Engineers' Handbook, 5th Ed.; Perry, R.H. and Chilton, C.H., eds.; McGraw-Hill; New York; 1973; p. 5-21; col. 2; p. 5-25, Table 2-13; and pp. 5-40 to 5-45.)

In sum, the Applicants respectfully request that these rejections be withdrawn.

CLAIM 24 IS NON-OBVIOUS

The rejection of claim 24 for obviousness, which the Applicants traverse, is believed moot in view of the above remarks and the novelty of parent claim 1.

DOUBLE PATENTING

Claim 1 stands provisionally rejected on the ground of non-statutory obviousness-type double patenting over claim 1 of co-pending Application No. 11/177,505 (2006/0193748) which is a related case and subject to examination. The Applicants propose to address this rejection when, if ever, it becomes a non-provisional rejection.

CONCLUDING REMARKS

The Applicants believe that the present application is in condition for allowance. Favorable reconsideration of the application as amended is respectfully requested.

The Examiner is invited to contact the undersigned by telephone if it would advance the prosecution of the present application.

The Commissioner is hereby authorized to charge any additional fees which may be required regarding this application under 37 C.F.R. §§ 1.16-1.17, or credit any overpayment, to Deposit Account No. 19-0741. Should no proper payment be enclosed herewith, as by the credit card payment instructions in EFS-Web being incorrect or absent, resulting in a rejected or incorrect credit card transaction, the Commissioner is authorized to charge the unpaid amount to Deposit Account No. 19-0741. If any extensions of time are needed for timely acceptance of

papers submitted herewith, Applicant hereby petitions for such extension under 37 C.F.R. §1.136 and authorizes payment of any such extensions fees to Deposit Account No. 19-0741.

Respectfully submitted,

Date June 17, 2009

By

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Chemical Engineers' Handbook

FIFTH EDITION

Prepared by a staff of specialists
under the editorial direction of

Robert H. Perry
Consultant

Cecil H. Chilton
Senior Advisor
Battelle Memorial Institute

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In the transition region (N_{Re} from 2000 to 3000), the velocity profile becomes more blunt and the ratio V/u_{max} increases (see Fig. 5-12). Velocity profile curves are given by Patel and Head [*J. Fluid Mech.*, 38, part 1, 181–201 (1969)] for flow in smooth pipes in the Reynolds number range of about 1500 to 10,000. At higher Reynolds numbers, the flow is generally fully turbulent, and the velocity profile in **smooth-wall pipes** is characterized by a laminar boundary layer ($y^+ < 5$), a turbulent core ($y^+ > 30$), and a buffer layer in between. The local velocity is given by the following relationships:

For the laminar boundary layer,

$$u^+ = y^+ \qquad \text{for } y^+ < 5 \tag{5-51a}$$

For the buffer layer,

$$u^+ = -3.05 + 5.00 \ln y^+ \qquad \text{for } 5 < y^+ < 30 \tag{5-51b}$$

For the turbulent core,

$$u^+ = 5.5 + 2.5 \ln y^+ \qquad \text{for } y^+ > 30 \tag{5-51c}$$

For **rough-wall pipes**, the local velocity in the turbulent core is given by

$$u^+ = 8.5 + 2.5 \ln \frac{y}{e} \qquad \text{for } y^+ > 30 \tag{5-51d}$$

where $u^+ = u/u^*$; u = local velocity, ft./sec., at distance y ft. from the pipe wall; $u^* = \sqrt{\tau_0 g_c/\rho}$ (called friction velocity); τ_0 = wall shear stress ($D\,\Delta p/4L$), lb. force/sq. ft.; g_c = dimensional constant, 32.17 (lb.)(ft.)/(lb. force)(sec.²); ρ = fluid density, lb./cu. ft.; Δp = pressure drop, lb. force/sq. ft.; D = inside pipe diameter, ft.; L = pipe length, ft.; $y^+ = yu^*\rho/\mu$, dimensionless; μ = fluid viscosity, lb./(ft.)(sec.); e = height of wall roughness, ft. For further details, see Knudsen and Katz, *op. cit.*, pp. 154–169, and Cremer and Davies, *op. cit.*, vol. 4, p. 401.

Equations describing the distribution of residence time for turbulent flow in pipes are given by Danckwerts, *loc. cit.*

Velocity Distribution, Other Shapes. For velocity profiles under laminar- and turbulent-flow conditions in annuli, between infinite parallel planes, and in other non-circular cross sections, see Knudsen and Katz, *op. cit.*; Purday, "Mechanics of Viscous Flow," Chap. II, Dover, New York, 1949; Rouse, "Advanced Mechanics of Fluids," p. 219, Wiley, New York, 1959; Goldstein, "Modern Developments in Fluid Dynamics," vol. 2, pp. 359–360, Oxford, London, 1938.

Analytically derived equations are presented by Straub, Silberman, and Nelson [*Trans. Am. Soc. Civil Engrs.*, 123, 685–714 (1958)] for **laminar flow** through a variety of open-channel cross sections, including semicircular, rectangular, triangular, elliptical, trapezoidal, etc.

Experimentally determined velocity profiles are also presented by Straub *et al.* for **turbulent flow** in triangular troughs. Profiles for channels of various cross sections are given in O'Brien and Hickox, "Applied Fluid Mechanics," pp. 268–270, McGraw-Hill, New York, 1937, and Chow, "Open-channel Hydraulics," pp. 24–29, McGraw-Hill, New York, 1959.

Residence-time Distribution, Process Vessels. An extensive treatment of distribution of residence time and of dispersion in a variety of typical process vessels is given by Levenspiel and Bischoff, "Patterns of Flow in Chemical Process Vessels," in Drew, Hoopes, and Vermeulen, "Advances in Chemical Engineering," vol. 4, Academic, New York, 1963. The case of multiple stirred tanks in series is covered in detail by Stokes and Nauman [*Can. J. Chem. Eng.*, 48, 723–725 (1970)]. Information on residence time and fluid mixing on commercial-scale sieve trays is given by Bell [*Am. Inst. Chem. Engrs. J.*, 18, 498–505 (1972)].

Incompressible Flow. The flow can be considered to be incompressible if (1) the substance flowing is a liquid or (2) if it is a gas whose density changes within the system no more than 10 per cent. In this event, if the inlet density is employed, the resulting error in computed pressure drop will generally not exceed the uncertainty limits in the friction factor. In the event of larger changes in fluid density, *e.g.*, gases with large pressure drops, the more exact methods described under Compressible Flow (pp. 5–26 to 5–31) should be used.

General Formulas and Methods. The problem of finding one of the three quantities—rate of discharge, size of channel, pressure or head loss—when the other two are given is solved by substituting the data of the problem in an appropriate form of the mechanical energy balance (p. 5–18) after the term F, frictional loss of mechanical energy, has been evaluated. That part of F which arises from friction within the channel proper is considered below. The part due to fittings, bends, and the like, which often constitutes a major part of the friction, is discussed on pp. 5–32 to 5–38.

The **Fanning, or Darcy, equation**, Eq. (5-52), for steady flow in **uniform circular pipes** running full of liquid under **isothermal conditions**

$$F = \left(\frac{4fL}{D}\right)\frac{V^2}{2g_c} = \left(\frac{4fL}{D}\right)h_v = \left(\frac{4fL}{D}\right)\frac{G^2}{2g_c\rho^2} = \frac{32fLw^2}{\pi^2\rho^2 g_c D^5} = \frac{32fLq^2}{\pi^2 g_c D^5} \tag{5-52}$$

gives the friction loss F in (ft.)(lb. force)/lb. of fluid flowing (or ft. of fluid flowing), where D = duct diameter, ft.; L = duct length, ft.; ρ = fluid density, lb./cu. ft.; V = fluid velocity, ft./sec.; h_v = velocity head ($V^2/2g_c$), ft. of fluid flowing; G = mass velocity, lb./(sec.)(sq. ft.); w = weight rate of flow, lb./sec.; q = volumetric rate of flow, cu. ft./sec.; g_c = dimensional constant, 32.17 (lb.)(ft.)/(lb. force)(sec.²); f = **Fanning friction factor** (see below), dimensionless.

The pressure drop due to friction is $\Delta p = F\rho$, lb. force/sq. ft.

The **Fanning friction factor** f is a function of the Reynolds number N_{Re} and the roughness of the channel inside surface ϵ. One widely used correlation [Moody, *Trans. Am. Soc. Mech. Engrs.*, 66, 671–684 (1944)], as shown in Fig. 5-26, is a plot of Fanning friction factor as a function of Reynolds number and relative roughness ϵ/D or ϵ''/D'', where ϵ = surface roughness, ft.; D = pipe inside diameter, ft.; ϵ'' = surface roughness, in.; and D'' = pipe inside diameter, in. Values of ϵ or ϵ'' for various materials are given in Table 5-7. Substitution of the equation for curve A, Fig. 5-26, into Eq. (5-52) yields Poiseuille's law for laminar flow ($N_{Re} \leq 2000$); see Table 5-13. Care must be exercised when values of f are taken from the literature, because the same name and symbol are sometimes used to denote various multiples of the f given by Fig. 5-26.

A rapid method of solving Eq. (5-52) for turbulent flow ($N_{Re} > 2000$) is to use the **alignment** chart in Fig. 5-27, which is based on curve D of Fig. 5-26 [Genereaux, *Chem. & Met. Eng.*, 44, 241–248 (1937)]. If, for the value of N_{Re}, obtaining some other value of f, say f', is preferred, the quantity sought, as given by the chart, should be multiplied by the factors given in Table 5-8. A nomograph to find pressure drop, taking into account pipe surface roughness, and a nomograph to determine flow rate or pipe size if either is unknown and if the pressure drop is known, are given by Arnold [*Chem. Eng.*, 66(11), 103–106 (1959)].

Table 5-7. Values of Surface Roughness for Various Materials*

Material	Surface roughness ϵ, ft.	Surface roughness ϵ'', in.
Drawn tubing (brass, lead, glass, and the like)	0.000005	0.00006
Commercial steel or wrought iron	0.00015	0.0018
Asphalted cast iron	0.0004	0.0048
Galvanized iron	0.0005	0.006
Cast iron	0.00085	0.010
Wood stave	0.0006–0.003	0.0072–0.036
Concrete	0.001–0.01	0.012–0.12
Riveted steel	0.003–0.03	0.036–0.36

*Moody, *Trans. Am. Soc. Mech. Engrs.*, 66, 671–684 (1944); *Mech. Eng.*, 69, 1005–1006 (1947). Additional values of ϵ for various types or conditions of concrete, wrought iron, welded steel, riveted steel, and corrugated metal pipes are given in King and Brater, "Handbook of Hydraulics," 5th ed., pp. 6-11 and 6-12, McGraw-Hill, New York, 1963.

rmula,*

	n
. . . .	0.014
. . . .	0.017
. . . .	0.013
. . . .	0.015
. . . .	0.012
. . . .	0.012
. . . .	0.013
. . . .	0.028
. . . .	0.023
. . . .	0.025
. . . .	0.028
. . . .	0.030
. . . .	0.040
. . . .	0.055
. . . .	0.070

. p. 7-17, McGraw- ow, "Open-channel 9.

experiments with ly given in terms ee Chow, "Open- York, 1959. Hen- New York, 1966;

(5-55)

ent (= $\sqrt{2g_c/f}$), us, ft.; S = slope also = F/L. The . formula:

(5-56)

For the turbulent s, Eq. (5-52) and mputed from the of fluids in open commended. For nels, V-troughs of lal channels, and elson [*Trans. Am.* critical Reynolds laminar and tur- her than that for , depending upon Owen, *Trans. Am.* the flow of non- and Tiu [*Can. J.*

/sq. in. gage and a /hr. through a 2-in. of pipe? air is (120 + 14.7)/ 5 and extend the line)°C. on the gas-tem- sect the $\mu_c^{0.16}/\rho$ line cting the $\Delta p'_p(P_a)/L$).00087 lb. force/(sq.

ine is to be pumped °C. If the allowable what size pipe is re-

i the intersection of . Extend the line to

Table 5-13. Laminar-flow Formulas

Ducts Running Full

For liquids, let $N = \frac{\rho g_c}{\mu}\left(\frac{g}{g_c}\rho \sin\alpha + \frac{p_1 - p_2}{L}\right)$; for gases, let $N^{(a)} = \frac{g_c M}{2zRT\mu}\left(\frac{p_1^2 - p_2^2}{L}\right)$

Duct cross section	Theoretical equation for weight rate of flow
Circle,[b],[c] diam. = D	$w = \frac{\pi D^4 N}{128}$ [for liquids this reduces to $p_1 - p_2 = \frac{32\mu LV}{g_c D^2}$ (*i.e.*, Poiseuille's law) if tube is horizontal]
Ellipse,[c] semiaxes = a,b	$w = \frac{\pi a^3 b^3}{a^2 + b^2}\left(\frac{N}{4}\right)$
Rectangle,[d] width = a, height = b	$w = \frac{ab^3 N}{K}$
where a/b = 1, 2, 3, 4, 5, 10, ∞ (broad parallel plates); K = 28.6, 17.5, 15.3, 14.2, 13.7, 12.8, 12	
Broad parallel plates,[b],[d] spacing = b; *i.e.*, rectangle with $a/b = \infty$	$w = \frac{b^3 N}{12}$ per unit width
Annulus,[c] outer diam. = D_2, inner diam. = D_1	$w = \frac{\pi(D_2^2 - D_1^2)N}{128}\left[D_2^2 + D_1^2 - \frac{D_2^2 - D_1^2}{2.3\log_{10}(D_2/D_1)}\right]$

Open Channels

Let $N = \frac{\rho^2 g_c \sin\alpha}{\mu}$, since necessarily $p_1 = p_2$; the following equations are valid only when the variation in depth is negligible

Channel cross section	Theoretical equation for weight rate of flow
Rectangle,[e] width = a, depth = $\frac{b}{2}$	$w = \frac{ab^3 N}{2K}$
where a/b = 1, 2, 3, 4, 5, 10, ∞ (broad stream); K = 28.6, 17.5, 15.3, 14.2, 13.7, 12.8, 12	
Broad stream on flat plate,[e] depth = $\frac{b}{2}$	$w = \frac{b^3 N}{24}$ per unit breadth
V-trough,[f] vertical ∠ = 90 deg., bisector vertical, slant depth = a	$w = \frac{a^4 N}{57}$

Notation used in Table 5-13:

a, b, D = characteristic lengths, ft.
g = local acceleration due to gravity, 32.2 ft./sec.^{2}
g_c = dimensional constant, 32.17 (lb.)(ft.)/(lb. force)(sec.^{2})
L = length of passage, ft.
M = molecular weight, lb./lb.-mole
p_1, p_2 = upstream and downstream static pressures, lb. force/sq. ft. abs.
R = gas constant, 1546 (ft.)(lb. force)/(°R.) (lb.-mole)
T = absolute temperature, °R. (°F. + 460)
w = weight rate of flow, lb./sec.
z = compressibility factor, dimensionless
α = angle between duct axis and horizontal, deg.
ρ = fluid density, lb./cu. ft.
μ = absolute viscosity, lb./(ft.)(sec.)

[a] If the pressure drop is less than 10 per cent of the downstream absolute pressure, the approximate expression $N = \frac{\rho g_c(p_1 - p_2)}{\mu L}$ may be used in case of gases.

[b] Dryden, Murnaghan, and Bateman, "Hydrodynamics," pp. 178, 184–185, Dover, New York, 1956.
[c] Lamb, "Hydrodynamics," 6th ed., p. 587, Cambridge, New York, 1932.
[d] Purday, "An Introduction to the Mechanics of Viscous Flow," pp. 16–18, Dover, New York, 1949.
[e] Owen, *Trans. Am. Soc. Civil Engrs.*, 119, 1157–1175 (1954).
[f] Straub *et al.*, *Trans. Am. Soc. Civil Engrs.*, 123, 685–714 (1958).

$\mu_c^{0.16}/\rho = 0.0179$ and connect that point to $\Delta p'_p/L = 0.006$ and extend to the reference line at point $B = 11.65$. Connect point B through $w' = 147$ (since at density of 73.3 lb./cu. ft., 250 gal./min. = 147,000 lb./hr.) to intersect at $D'_i = 5.5$ in., indicating a 6-in. pipe.

Allowance for Viscosity and Density. Gases. Temperature and molecular-weight scales are given in Fig. 5-27 in the form of a line-coordinated chart by which values of $\mu_c^{0.16}/\rho$ at atmospheric pressure are determined directly. Though the viscosities of gases and gas mixtures are not exactly proportional to molecular weight, the error is small because viscosity enters the calculation to only the 0.16 power.

Liquids. A separate temperature scale and grid are given on the chart. Coordinates given in Table 5-10 locate the point for a given liquid on the grid; a line through the point and the given temperature determines $\mu_c^{0.16}/\rho$ directly. Coordinates for liquids not given in the table may be determined by calculating values

wall of the cylindrical bob is computed from

$$\tau_i = \frac{2T}{\pi D_i^2 h} \tag{5-113}$$

where τ_i = shear stress at wall, lb. force/sq. ft.; T = torque, (lb. force)(ft.); D_i = diameter of bob, ft.; h = height of bob, ft. The shear rate at the wall of a cylindrical bob is given as [Krieger and Maron, *J. Appl. Phys.*, **25**, 72–75 (1954), see also Metzner in Drew and Hoopes, *loc. cit.*]

$$\left(-\frac{du}{dr}\right)_i = \frac{4\pi N}{1-(1/s^2)}\left[1 + k_1\left(\frac{1}{n''}-1\right) + k_2\left(\frac{1}{n''}-1\right)^2\right] \quad \text{for } s < 1.2 \tag{5-114}$$

$$k_1 = \frac{s^2-1}{2s^2}\left(1 + \frac{2}{3}\ln s\right) \tag{5-115}$$

and

$$k_2 = \frac{s^2-1}{6s^2}\ln s \tag{5-116}$$

where $(-du/dr)_i$ = shear stress at wall of bob, 1/sec.; N = rotational speed, rev./sec.; n'' = slope of logarithmic plot of torque vs. rotational speed; s = ratio of cup diameter to bob diameter, dimensionless. From Eq. (5-102) values of K and n are obtained. Metzner and Reed [*Am. Inst. Chem. Engrs. J.*, **1**, 434–440 (1955)] have shown that for many fluids,

$$n' = n \tag{5-117}$$

and

$$K' = K\left(\frac{3n'+1}{4n'}\right)^{n'} \tag{5-118}$$

Discussions of theories for various viscometers are given by Oka (Chap. 2 of Eirich, "Rheology," vol. 3, Academic, New York, 1960); Van Wazer *et al.* ("Viscosity and Flow Measurement," Interscience, New York, 1963); and Wohl [*Chem. Eng.*, **75**(7), 99–104 (Mar. 25, 1968)]. Practical aspects of viscometry are given by Bowen [*Chem. Eng.*, **68**(17), 119; (18), 131 (1961)].

The applications of non-Newtonian flow theory to operations in **polymer processing**, such as mixing, extrusion, calendering, fiber spinning, and sheet forming, are described in Bernhardt, "Processing of Thermoplastic Materials," Reinhold, New York, 1959; Eirich, *loc. cit.*; McKelvey, "Polymer Processing," Wiley, New York, 1962; and Wilkinson, "Non-Newtonian Fluids," Pergamon, New York, 1960.

TWO-PHASE FLOW

Liquids and Gases. For cocurrent flow with constant liquid-gas ratios, considerable experimental and theoretical work has been done on prediction of pressure drop, volume fractions, and flow pattern for flow in pipes. A reliable general correlation has not as yet been developed, although correlations for specific flow systems have been published. Presented here are guides for the estimation of flow pattern, pressure drop and volume fractions for flow in horizontal and vertical pipes.

In **horizontal pipe**, flow patterns have been reported in the literature and correlated empirically as functions of flow rates and flow properties. The boundaries between flow patterns, however, are not sharply defined, because the transitions are gradual and the mean boundaries depend upon interpretations of individual investigators and upon piping configurations and fluids under study. The following general types of **flow pattern** have been reported, where the values of the superficial velocities given are representative values for liquids with viscosities less than about 100 centipoise and gases of densities about that of air:

1. **Bubble or froth flow**, in which bubbles of gas are dispersed throughout the liquid, occurs for liquid superficial velocities from about 5 to 15 ft./sec. and gas superficial velocities from about 1 to 10 ft./sec. (See p. 5-4 for definition of superficial velocity.)

2. **Plug flow**, in which alternate plugs of liquid and gas move along the upper part of the pipe, occurs for liquid superficial velocities less than 2 ft./sec. and gas superficial velocities less than 3 ft./sec.

3. **Stratified flow**, in which the liquid flows along the bottom of the pipe and the gas flows over a smooth liquid-gas interface, occurs for liquid superficial velocities less than 0.5 ft./sec. and gas superficial velocities from about 2 to about 10 ft./sec.

4. **Wavy flow** is similar to stratified flow except the interface has waves traveling in the direction of flow. This occurs for liquid superficial velocities less than about 1 ft./sec. and gas superficial velocities about 15 ft./sec.

5. **Slug flow**, in which a wave is picked up periodically by the rapidly moving gas to form a frothy slug which passes along the pipe at a greater velocity than the average liquid velocity. In this type of flow, slugs can cause severe and, in some cases, dangerous vibrations in equipment because of impact of the high-velocity slugs against such fittings as return bends.

6. **Annular flow**, in which the liquid flows as a film around the pipe inside wall and the gas flows as a core. A portion of the liquid is entrained as a spray by the central gas core. This type of flow occurs for gas superficial velocities greater than about 20 ft./sec. Determinations of entrainment are reported by Wicks and Dukler [*Am. Inst. Chem. Engrs. J.*, **6**, 463–468 (1960)], and Magiros and Dukler, pp. 532–553, in Lay and Malvern, "Developments in Mechanics," vol. 1, Plenum Press, New York, 1961.

7. **Spray or dispersed flow**, in which nearly all the liquid is entrained as fine droplets by the gas, probably occurs for gas superficial velocities greater than 200 ft./sec. The interaction between an air stream and a moving horizontal water surface is described by Hanratty and Engen [*Am. Inst. Chem. Engrs. J.*, **3**, 299–304 (1957)].

The flow pattern may be predicted from the correlation proposed by Baker [*Oil Gas J.*, **53**(12), 185–190, 192, 195 (1954)] as shown in Fig. 5-50. In this figure,

$$\lambda = \left[\left(\frac{\rho_G}{0.075}\right)\left(\frac{\rho_L}{62.3}\right)\right]^{1/2} \tag{5-119}$$

$$\psi = \frac{73}{\sigma'}\left[\mu_L'\left(\frac{62.3}{\rho_L}\right)^2\right]^{1/3} \tag{5-120}$$

where G_G = gas mass velocity, lb./(sec.)(sq. ft.); G_L = liquid mass velocity, lb./(sec.)(sq. ft.); μ_L' = liquid viscosity, centipoise; ρ_G = gas density, lb./cu. ft.; ρ_L = liquid density, lb./cu. ft.; σ' = liquid surface tension, dynes/cm.

For additional details and references on flow pattern, see Anderson and Russell [*Chem. Eng.*, **72**(25), 135–144 (Dec. 6, 1965)]; Dukler and Wicks (in Acrivos, "Modern Chemical Engineering," vol. I, Chap. 8, Reinhold, New York, 1963); and Scott (in Drew *et al.*,

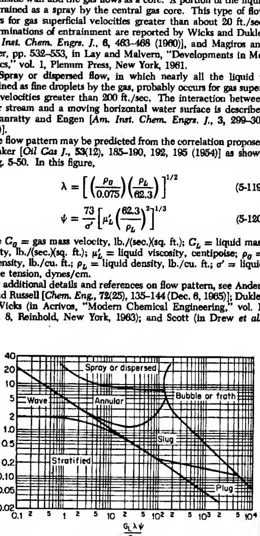

FIG. 5-50. Flow-pattern regions in cocurrent liquid-gas flow through horizontal pipes. [*From Baker, Oil Gas J.*, **53**, *(12), 185–190, 192, 195 (1954).*]

"Advances in Chemical Engineering," vol. 4, pp. 199–277, Academic, New York, 1963).

Two-phase pressure drop due to friction for cocurrent flow in **horizontal pipe** can be estimated by the semiempirical correlation of Lockhart and Martinelli [*Chem. Eng. Progr.*, **45**, 39–48 (1949)]. Basis of the correlation is that the two-phase pressure drop is equal to the single-phase pressure drop for either phase multiplied by a factor found to be a function of the single-phase pressure drops of the two phases:

$$\left(\frac{\Delta p}{\Delta L}\right)_{TP} = Y_L \left(\frac{\Delta p}{\Delta L}\right)_L \tag{5-121}$$

or

$$\left(\frac{\Delta p}{\Delta L}\right)_{TP} = Y_G \left(\frac{\Delta p}{\Delta L}\right)_G \tag{5-122}$$

where

$$Y_L = F_1(X) \qquad \text{and} \qquad Y_G = F_2(X) \tag{5-123}$$

$$X = \left[\frac{(\Delta p/\Delta L)_L}{(\Delta p/\Delta L)_G}\right]^{1/2} \tag{5-124}$$

Note that

$$Y_G = X^2 Y_L \tag{5-125}$$

The single-phase pressure-drop gradients $(\Delta p/\Delta L)_L$ and $(\Delta p/\Delta L)_G$ are computed from the Fanning equation, Eq. (5-52), assuming that each phase is flowing alone in the pipe; that is, superficial velocities are used. The superficial velocities are based on the full cross-sectional area of the pipe:

$$V_L = \frac{q_L}{A} \qquad \text{and} \qquad V_G = \frac{q_G}{A} \tag{5-126}$$

where V_L = liquid-phase superficial velocity, ft./sec.; V_G = gas-phase superficial velocity, ft./sec.; q_L = liquid-phase volumetric flow rate, cu. ft./sec.; q_G = gas-phase volumetric flow rate, cu. ft./sec.; A = pipe cross-sectional area, sq. ft.

Functions F_1 and F_2, Eq. (5-123), are shown as curves in Fig. 5-51. Separate curves are required for each flow regime; liquid in viscous* flow and gas in viscous flow (*vv*); liquid viscous-gas turbulent (*vt*); and similarly (*tv*) and (*tt*). In Fig. 5-51, however, only one curve is given for liquid viscous-gas turbulent and liquid turbulent-gas viscous because the difference between the experimental curves is

*The term "viscous" is customarily used, rather than "laminar," in reference to two-phase flow.

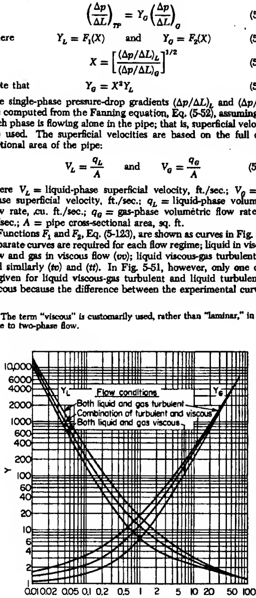

FIG. 5-51. Parameters for pressure drop in liquid-gas flow through horizontal pipes. [*Based on Lockhart and Martinelli, Chem. Eng. Progr.*, **45**, 39 (1949).]

small compared with the uncertainty of the correlation. The transition criterion between viscous flow and turbulent flow is not definitely known. However, for design purposes, the transition can be taken as that for single-phase flow; that is, the flow can be considered viscous for $N_{Re} \leq 2000$ and turbulent for $N_{Re} > 2000$, where N_{Re} is based on superficial velocity. Lockhart and Martinelli (*loc. cit.*) correlated their pressure-drop data on flows in pipes 1 in. or less in diameter within about ±50 per cent. In general, the predictions are high for stratified, wavy, and slug flows and low for annular flow. The correlations can be applied to pipe diameters up to 4 in. with about the same accuracy. Several investigators have studied flows in larger pipes (up to 10 in.) and have developed equations for pressure drop in their particular system; however, a better general correlation has not been developed. For references to other correlations, see Dukler and Wicks in Acrivos, *loc. cit.*; Scott in Drew *et al.*, *loc. cit.*; DeGance and Atherton [*Chem. Eng.*, **77**(15), 95–103 (1970)].

Volume fraction or **hold up** of a phase for cocurrent flow in **horizontal pipe** can be predicted from the following correlation developed by Lockhart and Martinelli (*loc. cit.*):

$$R_L = F_3(X) \qquad \text{and} \qquad R_G = F_4(X) \tag{5-127}$$

where

$$R_L + R_G = 1 \tag{5-128}$$

R_L = fraction of pipe volume occupied by the liquid phase, dimensionless; R_G = fraction occupied by gas phase, dimensionless; X = parameter defined by Eq. (5-124), dimensionless. Function F_3 as a curve of R_L vs. X is given in Fig. 5-52. Lockhart and Martinelli correlated data for pipe diameters 1 in. and less within ±50 per cent of the curve shown. Indications are that liquid-volume fractions may be less than those predicted by Fig. 5-52 for liquid viscosities greater than 1 centipoise [see Alves, *Chem. Eng. Progr.*, **50**, 449–456 (1954)] and greater than predicted for larger pipe diameters (see Baker, *loc. cit.*).

Pressure-drop data for a 1-in. **inlet or feed tee**, with the liquid entering the run and the gas entering the branch, are given by Alves (*loc. cit.*). For **fittings** and **valves**, results by Chenoweth and Martin [*Petrol. Refiner*, **34**(10), 151–155 (1955)] indicate that single-phase data can be used in their correlation for two-phase pressure drop. For **orifices**, correlations are given by Murdock [*J. Basic Eng.*, **84**, 419–433 (1962)] and Chisholm [*Brit. Chem. Eng.*, **12**, 1368–1371 (1967)].

For upflow in vertical pipe, considerable work has been done on gas lifts, a type of liquid pump. A gas lift consists simply of a vertical pipe, open at both ends, part of which is submerged below the surface of the liquid to be pumped. Compressed gas is admitted to the bottom of the pipe, forming a mixture of liquid and gas within the pipe. The gas reduces the average density of the mixture to a value where the weight of the mixture is less than equivalent to the submergence or pressure at the air inlet. Thus at the proper rates of liquid and gas, the mixture rises upward through the pipe and is discharged at the upper end. The submergence is the distance from the liquid surface to the air inlet. The lift is the distance from the liquid surface to the discharge. The submergence ratio is defined as

$$R_s = \frac{\text{submergence}}{\text{submergence} + \text{lift}} \tag{5-129}$$

For **upflow** the following general types of **flow pattern** have been observed where the values of the superficial velocities given are representative for liquids with viscosities less than about 100 centipoise and gas densities about that of air:

1. **Bubble or aerated flow**, in which the gas is dispersed as fine bubbles throughout the liquid, occurs for gas superficial velocities below about 2 ft./sec.

2. **Piston, plug, or slug flow**, in which the gas flows as large plugs, occurs for gas superficial velocities from about 2 to about 30 ft./sec.

3. **Froth flow**, in which the gas bubbles mix with the liquid in a highly turbulent pattern.

4. **Ripple or wave flow**, in which there is an upward-moving wavy layer of liquid on the wall.

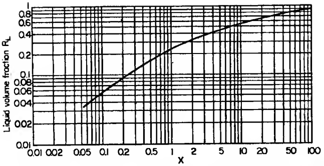

FIG. 5-52. Liquid volume fraction in liquid-gas flow through horizontal pipes. [*Lockhart and Martinelli, Chem. Eng. Progr.*, **45**, 39 (1949).]

5. Annular or film flow, in which the liquid flows up the pipe as an annulus and the gas flows as a core, occurs for liquid superficial velocities less than 2 ft./sec. and gas superficial velocities over 30 ft./sec. Data on entrainment are presented by Gill, Hewitt, and Lacey [*Chem. Eng. Sci.*, **20**, 71–88 (1965)].

6. **Mist flow**, in which the liquid is carried as fine drops by the gas phase. Data indicate that this probably occurs for superficial gas velocities over about 70 ft./sec. [Collier and Hewitt, *Trans. Inst. Chem. Engrs. (London)*, **39**, 127–136 (1961); Yagi and Sasaki, *Chem. Eng. (Japan)*, **17**, 216–223 (1953)].

The flow pattern may be predicted from the correlation proposed by Govier, Radford, and Dunn [*Can. J. Chem. Eng.*, **35**, 58–70 (1957)] as shown in Fig. 5-53. In this figure,

V_L = superficial liquid velocity, ft./sec.
R_V = ratio of volumetric flow rates of gas to liquid entering pipe, dimensionless

For additional details or references on flow pattern, see Anderson and Russell (*loc. cit.*); Gosline [*Trans. Am. Inst. Mining Met. Engrs. (Petrol. Develop. Technol.)*, **118**, 56–70 (1936)]; Huntington *et al.* [*Trans. Am. Inst. Mining Met. Engrs. (Petrol. Develop. Technol.)*, **136**, 79–90 (1940); *Petrol. Refiner*, **33**(11), 208–211 (1954)].

During upflow, there is considerable slippage between the liquid and the gas; thus the ratio of liquid to gas within the pipe is greater than that at the inlet. Experimental data for the flow of water-air and kerosene-air mixtures in ½- and 2-in. pipes are presented by Galegar, Stovall, and Huntington [*Petrol. Refiner*, **33**(11), 208–211 (1954)]; experimental data and a correlation for the flow of water-air mixtures in ½- to 2.5-in. pipes are given by Govier and Short [*Can. J. Chem. Eng.*, **36**, 195–202 (1958)] and by Brown, Sullivan, and Govier [*ibid.*, **38**, 62–66 (1960)]. A proposed correlation is presented by Hughmark [*Chem. Eng. Progr.*, **58**(4), 62–65 (1962)]. Some data on liquid entrainment are given by Anderson and Mantzouranis [*Chem. Eng. Sci.*, **12**, 233–242 (1960)].

For prediction of **pressure drop in upflow**, a proposed correlation together with references to other correlations is presented by Hughmark [*Ind. Eng. Chem. Fundamentals*, **2**, 315–321 (1963)]. The **performance of gas lifts** can be predicted by the theory presented by Stenning and Martin [*J. Eng. Power*, **90**, 106–110 (1968)] or from the data of Shaw (*Texas Eng. Expt. Sta. Bull.* 113, 1949).

For **upflow in helically coiled tubes**, the flow pattern, pressure drop, and hold-up can be predicted by the correlations of Banerjee, Rhodes, and Scott [*Can. J. Chem. Eng.*, **47**, 445–453 (1969)].

For **downflow** in vertical pipes, **pressure-drop** measurements for the **annular flow** of water-air mixtures in 1-in. pipe and for the condensation of organic vapors inside a vertical tube (0.459 in. inside diameter) condenser have been tentatively correlated by Bergelin, Kegel, Carpenter, and Gazley (*Proc. Heat Transfer and Fluid Mech. Inst.*, A.S.M.E., June 22–24, 1949, pp. 19–28) based on a modified Fanning friction factor as shown in Fig. 5-54. In this figure, f'_G = modified Fanning friction factor, dimensionless; $(N_{Re})_G$ = Reynolds number, dimensionless = $DV_G\rho_G/\mu_G$; D = inside diameter of pipe, ft.; V_G = gas superficial velocity, ft./sec. = q_G/A; q_G = gas volumetric flow rate, cu. ft./sec.; A = cross-sectional area of pipe, sq. ft.; ρ_G = gas density, lb./cu. ft.; μ_G = gas viscosity, lb./(ft.)(sec.); σ_W/σ_L = ratio of surface tension of water at its boiling point to that of other liquid at its boiling point,

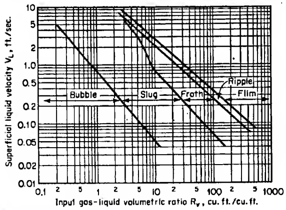

FIG. 5-53. Flow-pattern regions in cocurrent liquid-gas flow in upflow through vertical pipes. [*Govier, Radford, and Dunn, Can. J. Chem. Eng.*, **35**, 58–70 (1957).]

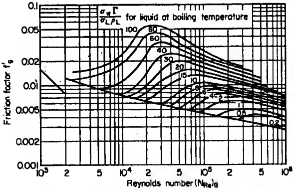

FIG. 5-54. Friction factors for condensing liquid-gas flow downward in vertical pipe. (*Bergelin et al., Proc. Heat Transfer and Fluid Mech. Inst., A.S.M.E., p. 19, 1949.*)

dimensionless; ρ_L = liquid density, lb./cu. ft.; Γ = liquid flow rate per unit pipe periphery, lb./(hr.)(ft.). Viscosity of the liquid is omitted from this correlation because in the experimental investigations the viscosities of the liquids used did not vary greatly; thus the effect of viscosity could not be determined. Pressure drop is computed as follows, using the Fanning equation based on the gas phase (effect of the liquid appears as a "roughness" in the friction factor):

$$\Delta p = \frac{4f'_G L \rho_G V_G^2}{2g_c D} \qquad (5\text{-}130)$$

where Δp = pressure drop, lb. force/sq. ft.; L = pipe length, ft.; g_c = dimensional constant, 32.17 (lb.)(ft.)/(lb. force)(sec.^{2}). Application of Eq. (5-130) to condensing flow in vertical condensers is described below, p. 5-45. A similar correlation for water-air flows in a 2-in. pipe is presented by Chien and Ibele [*J. Heat Transfer*, **86**, 89–96 (1964)].

For downflow in helically coiled tubes, some data on flow pattern, pressure drop, and hold-up are presented by Casper [*Chem. Ing. Tech.*, **42**, 349–354 (1970)].

Also involving downflow are drain and overflow pipes. The entrance to a drain pipe is flush with a horizontal surface; the entrance to an overflow pipe is above the horizontal surface, *i.e.*, the pipe extends through and above the horizontal surface. When such pipes do not run full, considerable amounts of gas can be drawn down by the flowing liquid. The amount of gas entrained is a function of pipe diameter, pipe length and liquid flow rate. Extensive data on air entrainment and head above the entrance as a function of water flow rate for pipe diameters from 1.73 to 5.84 in. and lengths from about 4 to 17 ft. are reported by Kalinske (*Univ. Iowa Studies in Engineering, Bull.* **26**, pp. 26–40, 1939–1940). For heads greater than the critical, the pipes will run full and no gas will be entrained. The critical head h for flow of water in drains and overflow pipes, as reported by Kalinske, is given as a function of pipe diameter D and length L in Fig. 5-55. From Kalinske's investigations, the height of protrusion of an overflow pipe apparently has no appreciable effect if the height of protrusion is greater than about one pipe diameter. For low heads, the liquid at the entrance to the pipe is generally swirling. The direction of swirl is theoretically counterclockwise (looking down) in the northern hemisphere. Practically, however, the direction of swirl will depend upon the approach conditions. For additional details, see Marris [*J. Appl. Mech.*, **34**, 11–15 (1967)]. Entrapped air bubbles can rise when the liquid velocity is

$$V_L \leq 0.31\sqrt{gD} \qquad (5\text{-}130a)$$

where V_L = liquid velocity, ft./sec.; g = acceleration due to gravity, approximately 32.2 ft./sec.^{2}; D = pipe inside diameter, ft. For additional information, see Simpson [*Chem. Eng.*, **75**(13), 192–214 (1968)].

For cocurrent flow with variable liquid-gas ratios, a large amount of theoretical and experimental information is available on flashing flow, and a limited amount is available on condensing flow.

Flashing flow occurs when a liquid which is initially saturated,

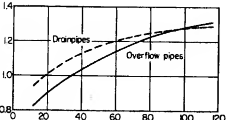

FIG. 5-55. Critical head for drain and overflow pipes. (*Kalinske, Univ. Iowa. Studies in Engineering, Bull. 26, 1939–1940.*)

i.e., at its boiling point, flows through a pipe. Several processes take place: (1) the pressure decreases because of friction, (2) the saturation temperature decreases because the pressure decreases, and (3) a portion of the liquid is vaporized. Thus two-phase flow occurs, with the ratio of the two phases continuously changing. Several types of flow are possible (see p. 5-40); however, in this case, vapor is being produced throughout the liquid, which probably tends to produce bubble flow. The flow patterns observed for the flow of an evaporating refrigerant in a horizontal pipe are described by Zahn [*J. Heat Transfer*, **86**, 417–429 (1964)].

In the following method for predicting flow conditions, the principal assumptions are that (1) liquid and vapor flow with equal velocities, (2) elevation changes and heat losses are negligible, and (3) the mixture behaves as a compressible fluid. The basic relation between flow conditions and pipe geometry, which can be derived from the continuity and energy equations [see Benjamin and Miller, *Trans. Am. Soc. Mech. Engrs.*, **64**, 657–669 (1942); Allen, *ibid.*, **73**, 257–265 (1951); Bridge, *Heating, Piping Air Conditioning*, **21**(5), 98–100 (1949); Mikol, *Trans. Am. Soc. Refrig. Air-Cond. Engrs.*, **69**, 213–225 (1963)] is

$$v_m\,dp + \frac{G_m^2}{g_c}\left(v_m\,dv_m + \frac{4fv_m^2\,dL}{2D}\right) = 0 \qquad (5\text{-}131)$$

Equation (5-131) can be integrated and solved graphically for the desired variable if the mathematical relation between mixture volume and pressure is known.

In general, Eq. (5-131) can be solved more conveniently by rewriting it and solving numerically:

$$\Delta L = -\frac{D}{2f}\left(\frac{g_c\,\Delta p}{G_m^2\bar{v}_m} + \frac{\Delta v_m}{\bar{v}_m}\right) \qquad (5\text{-}132)$$

where ΔL = increment of length, ft.; D = pipe diameter, ft.; f = Fanning friction factor, dimensionless; g_c = dimensional constant, 32.17 (lb.)(ft.)/(lb. force)(sec.^{2}); G_m = mass velocity of mixture, lb./(sec.)(sq. ft.); Δp = increment of pressure (downstream minus upstream pressures), lb. force/sq. ft.; $\bar{v}_m$ = average specific volume of mixture over Δp, cu. ft./lb. of mixture; Δv_m = incremental specific volume of mixture over Δp, cu. ft./lb. of mixture. The expansion is assumed to be an irreversible adiabatic (constant-enthalpy) process. Value of the friction factor f can be taken as 0.003, which is the average experimental value for the flow of a flashing mixture of water and steam determined by Benjamin and Miller (*loc. cit.*) and Bottomley [*Trans. North East Coast Inst. Engrs. & Shipbuilders*, **53**, 65–100 (1936)]. The value of the friction factor may be greater for flashing refrigerants in capillary tubes (see Mikol, *loc. cit.*).

In numerical integration of Eq. (5-132) for pipe length, the following general steps are taken: (1) mass velocity is known; (2) a pipe diameter is assumed; (3) increments of pressure are taken from the initial pressure to the final pressure; (4) average specific volume and the difference in specific volume of the mixture for each increment are obtained for a constant-enthalpy process, where the value of enthalpy is that of the saturated liquid at the initial pressure; (5) the increment of length for each increment of pressure taken is computed from Eq. (5-132), using the value of the friction factor given above; (6) total length of pipe is the sum of the length increments. Computations for the flow of water-steam mixtures in pipes are described by Bridge [*Heating, Piping Air Conditioning*, **21**(4), 92–96 (1949)] and Keister [*Petrol. Refiner*, **27**(11), 616–619 (1948)] and of refrigerants in capillary tubes by Mikol (*loc. cit.*). In this numerical integration, if the computed increment of length is negative, then the increment of pressure taken has been too large. A computed increment of length of zero is the condition for "critical flow" (see below for detailed discussion) and corresponds to the final discharge pressure. For cases where critical flow is indicated but where the computed discharge pressure is larger than desired or the pipe length is shorter, a larger pipe diameter should then be assumed and the integration repeated until the desired conditions are obtained. Solving Eq. (5-132) for pipe length when the other variables are known is comparatively straightforward; however, solving for one of the other variables involves the above numerical

integration on a trial-and-error basis. Charts for the maximum flow of water-steam mixtures in pipes, taking into account slip between the steam and the water, are presented by Moody [*J. Heat Transfer*, **88**, 285–295 (1966)].

The **critical flow** condition in **flashing flow** is similar to that in the single-phase flow of gases through pipes. When the discharge pressure is gradually decreased a pressure is reached at which further decrease in pressure will have no effect on pressure or flow rate within the pipe. The velocity at the discharge for these conditions is called the "critical velocity." For the single-phase flow of gases, this critical velocity is equal to the velocity of sound at the discharge pressure and temperature condition (see p. 5–4). For the flow of flashing mixtures, the critical velocity may be much less than the velocity of sound in the vapor phase.

The **maximum** possible **rate of flow** of a flashing mixture through a pipe of constant cross section, assuming that the mixture behaves as a compressible fluid, occurs when at the discharge end all the energy released by the differential drop in pressure is quantitatively converted into kinetic energy; *i.e.*, frictionless flow [see Lapple *et al.*, "Fluid and Particle Mechanics," p. 103, University of Delaware, Newark, 1951; Schweppe and Foust, *Chem. Eng. Progr.*, **49**, *Symp. Ser.* 5, 77–89 (1953)]:

$$(G_c)^2_m = -g_c\left(\frac{dp}{dv_m}\right) \qquad (5\text{-}133)$$

where $(G_c)_m$ = mixture critical mass velocity, lb./(sec.)(sq. ft.); g_c = dimensional constant, 32.17 (lb.)(ft.)/(lb. force)(sec.²); p = pressure, lb. force/sq. ft.; v_m = specific volume of mixture, cu. ft./lb. of mixture. To account for slip between the vapor and the liquid, v_m is evaluated from the following equations [see Levy, *J. Heat Transfer*, **87**, 53–58 (1965)]:

$$v_m = \frac{v_G x^2}{\alpha} + \frac{v_L(1-x)^2}{1-\alpha} \qquad (5\text{-}134)$$

$$\alpha = \left[1 + \frac{(1-x)(v_L/v_G)^{0.5}}{x}\right]^{-1} \qquad (5\text{-}135)$$

where v_G = specific volume of vapor, cu. ft./lb.; v_L = specific volume of liquid, cu. ft./lb.; x = vapor weight fraction, dimensionless; α = vapor volume fraction, dimensionless. The term dp/dv_m should be taken along a constant-entropy path for the conditions existing at the end of the pipe (for frictionless flow and reversible adiabatic expansion).

For the **critical flow of water-steam mixtures**, the maximum rate of flow and the fraction vaporized as functions of initial conditions and critical discharge pressures are given in Fig. 5-56 [Cruver and Moulton, *Am. Inst. Chem. Engrs. J.*, **13**, 52–60 (1967)]. In Fig. 5-56, G_c = critical or maximum mass velocity, lb./(sec.)(sq. ft.); E = initial enthalpy + initial kinetic energy + heat added, B.t.u./lb.; p_c' = absolute pressure inside tube at discharge end under critical flow conditions, lb. force/sq. in.; x = vapor weight fraction or fraction of liquid vaporized, dimensionless.

Flow of flashing mixtures of water and steam through valves is discussed by Allen [*Power Eng.*, **56**(5), 60–61, 102, 104, 106 (1952); **56**(6), 83–85 (1952); **57**(10), 94 (1953)]; through orifices and nozzles by Bailey [*Trans. Am. Soc. Mech. Engrs.*, **73**, 1109–1116 (1951)], Burnell [*Engineering*, **164**, 572–576 (1947)], and Murdock [*J. Heat Transfer*, **84**, 419–433 (1962)]; flow of carbon dioxide liquid and vapor through orifices and nozzles by Hesson and Peck [*Am. Inst. Chem. Engrs. J.*, **4**, 207–210 (1958)]; flow of refrigerant liquid and vapor through orifices and nozzles by Min, Fauske, and Patrick [*Ind. Eng. Chem. Fundamentals*, **5**, 50–55 (1966)] and Pasqua [*Refrig. Eng.*, **61**, 1084–1088, 1131 (1953)].

In flow of a **condensing vapor in horizontal straight circular pipe**, the pressure drop is the algebraic sum of the friction loss and the momentum changes. One general model [Soliman, Schuster, and Berenson, *J. Heat Transfer*, **90**, 267–276 (1968)] assumes that the condensate flows as an annular ring with the vapor as the core. The Lockhart-Martinelli pressure-drop correlation, p. 5-41, is recommended for predicting the friction loss. The momentum changes are estimated by the homogeneous-flow model [Andeen and Griffith, *J. Heat Transfer*, **90**, 211–222 (1968)], *i.e.*, uniform velocity profile,

$$(\Delta p_a)_{1-2} = \frac{G_m}{g_c}(V_{m2} - V_{m1}) \qquad (5\text{-}136)$$

where $(\Delta p_a)_{1-2}$ = pressure difference due to momentum changes between point 1 and point 2, lb. force/sq. ft.; G_m = mixture mass velocity, lb./(sec.)(sq. ft.); g_c = dimensional constant, 32.17 (lb.)(ft.)/(lb. force)(sec.²); V_{m2} = mixture average velocity at point 2, ft./sec.; V_{m1} = mixture average velocity at point 1, ft./sec. The pressure drop is computed for increments of pipe length commenc-

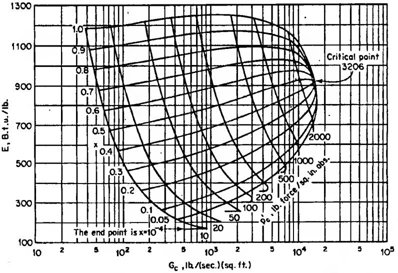

FIG. 5-56. Maximum discharge rates for water-steam mixtures inside pipes. [*Cruver and Moulton, Am. Inst. Chem. Engrs. J.*, 13, 52 (1967).]

ing at the inlet from a knowledge of the condensation rate with length. The total pressure drop is the sum of the pressure drops for the increments.

For **condensing vapor flowing vertically downward in a circular pipe**, the pressure drop can be estimated by use of the differential form of Eq. (5-130):

$$-\frac{dp}{dL} = \frac{4f'_G\rho_G V_G^2}{2g_cD} \tag{5-137}$$

From the heat-transfer characteristics the amount of vapor condensed as a function of pipe length can be computed. Then the pressure gradient $-dp/dL$ can be computed as a function of pipe length from Eq. (5-137) and Fig. 5-54. From a plot of pressure gradient vs. pipe length, the pressure drop is given by integration of the area under the curve. Bergelin *et al.* (*Proc. Heat Transfer and Fluid Mech. Inst.*, A.S.M.E., June 22–24, 1949, pp. 19–28) report considerable variation between experimental and predicted values; however, a more accurate method of prediction is not known. The over-all pressure drop across an actual condenser includes the entrance and header losses.

Gases and Solids. For the flow of gases and solids in a **horizontal pipe**, there are several modes of flow possible, depending upon density of the solids, solids-to-gas weight-rate ratio (loading), and gas velocity. With low-density solids or low solids-to-gas ratios, and high gas velocities, the solids may be fully suspended and fairly uniformly dispersed over the pipe cross section; with low solids-to-gas ratios and low gas velocities, the solids may bounce along the bottom of the pipe; with high solids-to-gas ratios and low gas velocities, the solids may settle to the bottom of the pipe and form dunes, with the particles moving from dune to dune, or form slugs, depending upon the nature of the particles. For additional details on modes of flow, see Coulson and Richardson (*op. cit.*, vol. 2, p. 538); Korn [*Chem. Eng.*, 57(3), 108–111 (1950)]; Patterson [*J. Eng. Power*, 81, 43–54 (1959)]; Wen and Simons [*Am. Inst. Chem. Engrs. J.*, 5, 263–267 (1959)].

No single correlation is available for prediction of the **minimum carrying velocity** for all solid-to-gas ratios in **horizontal pipe**. Guides are available, however, for estimation of carrying velocities. For low solids-to-gas weight-rate ratios, such as used in conventional pneumatic conveying, the minimum carrying velocity can be estimated by the following equation proposed by Dalla Valle [*Heating, Piping Air Conditioning*, 4, 639–641 (1932)], based on conveying tests with particles less than 0.32 in. in size and density less than 165 lb./cu. ft., using air as the carrying gas:

$$V_{c,h} = 270\left(\frac{\rho_s}{\rho_s + 62.3}\right)D_s^{0.40} \tag{5-138}$$

where $V_{c,h}$ = minimum carrying velocity, ft./sec.; ρ_s = density of the solids particles, lb./cu. ft.; D_s = diameter of largest particle to be conveyed, ft. Some data for single particles and mixed particle size are given by Zenz [*Ind. Eng. Chem. Fundamentals*, 3, 65–75 (1964)]. In practice, the actual conveying velocities used in systems with low solids-to-gas weight-rate ratios (<10), *i.e.*, conventional pneumatic conveyors, are generally over 50 ft./sec. [see Hudson, *Chem. Eng.*, 61(4), 191–194 (1954); Jorgensen, "Fan Engineering," 7th ed., p. 486, Buffalo Forge Co., Buffalo, 1970; Kunii and Levenspiel, "Fluidization Engineering," Chap. 12, Wiley, New York, 1969; and Zenz and Othmer, "Fluidization and Fluid-particle Systems," pp. 220–222, 322, Reinhold, New York, 1960]. For high solids-to-gas weight-rate ratios (>20), the actual gas velocities used are generally less than 25 ft./sec. and are approximately equal to twice the actual solids velocities (Wen and Simons, *loc. cit.*).

Total pressure drop in **horizontal pipe** may be considered as the sum of the following individual pressure drops [Mehta, Smith, and Comings, *Ind. Eng. Chem.*, 49, 986–992 (1957)]:

1. For acceleration of gas to the carrying velocity

$$\Delta p_{a,G} = \frac{G_G V_G}{2g_c} \tag{5-139}$$

2. For acceleration of solid particles

$$\Delta p_{a,s} = \frac{G_s V_s}{g_c} \tag{5-140}$$

3. For friction between gas and pipe wall

$$\Delta p_{f,G} = \frac{4f_G L\rho_{dG}V_G^2}{2g_cD_t} = \frac{4f_G L G_G V_G}{2g_cD_t} \tag{5-141}$$

4. For combined friction between particles and pipe wall, between gas and particles, and between particles, assuming that this friction can be expressed by a type of friction factor equation

$$\Delta p_{f,s} = \frac{4f_s L\rho_{ds}V_s^2}{2g_cD_t} = \frac{4f_s L G_s V_s}{2g_cD_t} \tag{5-142}$$

Friction factor f_s can be related to the particle drag coefficient by a force balance on the particles in the pipe as follows:

$$4f_s = \frac{3\rho_G D_t C}{2\rho_s D_s}\left(\frac{V_G - V_s}{V_s}\right)^2 \tag{5-143}$$

In the above equations, Δp = pressure drop, lb. force/sq. ft.; C = drag coefficient, dimensionless, obtained from Fig. 5-78, p. 5-62, for N_{Re} (Reynolds number) = $D_s(V_G - V_s)\rho_G/\mu_G$; D_s = diameter of solid particle, ft.; D_t = diameter of tube or pipe, ft.; f_G = Fanning friction factor, dimensionless, obtained from Fig. 5-26, p. 5-22; f_s = solids friction factor, dimensionless; $G_G = \rho_{dG}V_G$ = gas mass velocity, lb./(sec.)(sq. ft.); $G_s = \rho_{ds}V_s$ = solids mass velocity, lb./(sec.)(sq. ft.); g_c = dimensional constant, 32.17 (lb.)(ft.)/(lb. force)(sec.^{2}); L = length of pipe, ft.; V_G = actual velocity of gas, ft./sec.; V_s = actual velocity of solids, ft./sec.; ρ_{dG} = dispersed gas density, weight of gas/unit volume of pipe, lb./cu. ft.; ρ_{ds} = dispersed solids density, weight of solids/unit volume of pipe, lb./cu. ft.; ρ_G = gas density, lb./cu. ft.; ρ_s = solids density, lb./cu. ft.; μ_G = gas viscosity, lb./(ft.)(sec.).

For **solids-to-gas weight-rate ratios less than 5 in horizontal pipes**, such as those usually employed in conventional pneumatic conveying systems, Hinkle (Ph.D. Thesis, Chemical Engineering, Georgia Institute of Technology, Atlanta, 1953) found experimentally (for flow of air and solid particles 0.014 to 0.33 in. diameter in 2- and 3-in. glass pipe) that

$$V_s = V'_G\left[1 - 1.41D_s^{0.3}\left(\frac{\rho_s}{62.3}\right)^{0.5}\right] \tag{5-144}$$

where for this case $V'_G = G_G/\rho_G$ = superficial gas velocity, ft./sec. In such systems with low solids-to-gas ratios, $V_G \simeq V'_G$. For **solids-to-gas weight-rate ratios over 50**, such as those used in dense-phase transport, the particles tend to settle and move along the bottom of the pipe, and the flow pattern is therefore considerably different from that in the conventional dilute-phase pneumatic conveyor. Wen and Simons (*loc. cit.*) obtained the following empirical pressure-drop correlation for conditions where the particles were considered to have reached their terminal velocity. Their experiments involved the flow of glass beads (<0.01 in. diameter) and coal powder (<0.03 in. diameter) with air in a ¼-in. steel pipe and in ½- to 1-in. glass pipes.

$$\left(\frac{\Delta p_f}{L\rho_{ds}}\right)\left(\frac{D_t}{D_s}\right)^{0.25} = 2.5V_s^{0.45} \tag{5-145}$$

where Δp_f = sum of the pressure drops due to friction, lb. force/sq. ft.; L = length of pipe, ft.; $\rho_{ds} = G_s/V_s$ = dispersed solids density, weight of solids/unit volume of pipe, lb./cu. ft.; $G_s = w_s/A_t$ = superficial mass velocity of the solids, lb./(sec.)(sq. ft.); w_s = solids flow rate, lb./sec.; A_t = cross-sectional area of pipe, sq. ft.; D_t = diameter of pipe, ft.; D_s = diameter of solid particle, ft.; V_s = average actual velocity of solid particles, ft./sec., determined from Fig. 5-57. Average actual velocity of the air can be determined from the slippage between the air and the solids. Wen and Simons (*loc. cit.*) found that $V_a \simeq 2V_s$. Mass velocity of the air is given